Forest Hydrology: Lec. 12

Lecture content

- Evaporation
  - Diffusion and the Fick's law
  - Dalton law
  - Penman combination equation

Basic principle

Fick's Law:
A diffusing substance moves from where its concentration is larger to where its concentration is smaller at a rate that is proportional to the spatial gradient of concentration

indicates movement from regions of higher concentration to regions of lower concentration

$$F_z(X) = -D_X \frac{dC(X)}{dz}$$

gradient (change) in concentration

will have units of substance *[L T^{-1}]

where

$F_z(X)$ = rate of transfer of substance X in z direction

D_X = diffusivity of substance X [L^2 T^{-1}]

$C(X)$ = concentration of X ← **units depend on substance**

Evaporation

Fick's Law:
A diffusing substance moves from where its concentration is larger to where its concentration is smaller at a rate that is proportional to the spatial gradient of concentration

$$E = K_E v_a (e_s - e_a)$$

finite difference approx.

where

E = evaporation rate [$L\ T^{-1}$]

K_E = efficiency of vertical transport of water vapor [$L\ T^{-1}\ M^{-1}$]

v_a = wind speed [$L\ T^{-1}$]

e_s = vapor pressure of evaporating surface [$M\ L^{-1}\ T^{-2}$]

e_a = vapor pressure of overlying air [$M\ L^{-1}\ T^{-2}$]

Evaporation (mass transfer approach)

$$E = K_E v_a (e_s - e_a)$$

The mass-transfer approach makes direct use of this equation, often written as:

$$E = (b_0 + b_1 v_a)(e_s - e_a)$$

where b0 and b1 are empirical constants that depend chiefly on the height at which wind speed and air vapor pressure are measured. Relations of this form were first formulated by the English chemist John Dalton in 1802 and are often called Dalton-type equations.

Evaporation from a water surface (Penman combination approach) -1

If ground-heat conduction, water-advected energy and change in energy store can be neglected, the energy budget equation becomes:

$$E = \frac{R_n - H}{\rho_w \lambda_v} \qquad Eq.\ 1$$

where

- $\boldsymbol{E}$ = evaporation rate [L T^{-1}]
- $\boldsymbol{R_n}$ = net radiation
- $\boldsymbol{\lambda_v}$ = latent heat of vaporization
- $\boldsymbol{\rho_w}$ = density of water
- $\boldsymbol{H}$ = sensible heat transfer

Evaporation from a water surface (Penman combination approach) - 2

The sensible heat transfer flux is given by the following equation:

$$H = \frac{1}{r_{a,H}} \rho_a c_p \left(T_0 - T_z\right) \qquad \textit{Eq. 2}$$

where

H	= sensible heat flux
$r_{a,H}$	= aerodynamic resistance
ρ_a	= density of the air
c_p	= specific heat capacity of the air
T_0	= temperature at the surface
T_z	= temperature at height z

Evaporation from a water surface (Penman combination approach) - 3

The slope of the saturation-vapor-versus-temperature curve at the air temperature can be approximated by:

$$\Delta = \frac{e_s(T_0) - e_s(T_z)}{T_0 - T_z} \qquad \textit{Eq. 3}$$

$$\Rightarrow T_0 - T_z = \frac{e_s(T_0) - e_s(T_z)}{\Delta}$$

where

Δ = gradient of the saturation-vapor-versus temperature curve.

Evaporation from a water surface (Penman combination approach) - 4

Eq (3) can now be substituted into Eq. (2), as follows:

$$H = \frac{1}{r_{a,H}\Delta}\rho_a c_p \left[e_s(T_0) - e_s(T_z) \right] \qquad \textit{Eq. 4}$$

And this relation remains true if e_z (the vpd at elevation z) is added and subtracted from each of the terms in bracket:

$$H = \frac{1}{r_{a,H}\Delta}\rho_a c_p \left[e_s(T_0) - e_z \right] - \frac{1}{r_{a,H}\Delta}\rho_a c_p \left[e_s(T_z) - e_z \right] \qquad \textit{Eq. 5}$$

Evaporation from a water surface (Penman combination approach) - 5

Since

$$E = K_E v_a \left(e_s - e_z \right) = K_E v_a \left(e_s(T_0) - e_z \right) \quad Eq.\ 6$$

$$\Rightarrow e_s(T_0) - e_z = \frac{E}{K_E v_a}$$

Then substituting Eq. 6 and Eq. 1 into Eq. 5 yields Eq. 7, which represents the Penman Equation for evaporation

$$E = \frac{R_n + \dfrac{1}{r_{a,H}\Delta} \rho_a c_p \left(e_s(T_z) - e_z \right)}{\rho_w \lambda_v + \dfrac{1}{r_{a,H}\Delta} \rho_a c_p \dfrac{1}{K_E v_a}} \quad Eq.\ 7$$